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ON LAMBERT SERIES AND CONTINUED FRACTIONS

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Abstract: In this paper, an attempt has been made to establish certain results involving Lambert series and continued fractions.

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1. Introduction

It is now customary to define the basic hypergeometric series by

$$ {}_2\Phi_1[a,b;\,c;\,q;\,z] \;=\; {}_2\Phi_1\left[\begin{array}{l} a,b \quad ;q;\;z \\ c \end{array}\right] \;=\; \sum_{n=0}^{\infty} \frac{[a;q]_n\,[b;q]_n\;z^n}{[c;q]_n\;[q;q]_n}, \qquad (1.1)$$

where

$$[a;q] = \begin{cases} 1, & n=0, \\ (1-a)(1-aq)\dots(1-aq^{n-1}), & n=1,2,3,\dots \end{cases}$$

is the q-shifted factorial and it is assummed that $c \neq q^{-m}$ for $m = 0,1,2,\cdots$. Also, $|q| < 1$ and $|z| < 1$ for the converegence of the series (1.1).

The generalized bilateral basic hypergeometric series is defined by

$$ {}_r\Psi_r\left[\begin{array}{l} a_1,a_2,\cdots,a_r \quad ;q;\;z \\ b_1,b_2,\cdots,b_r \quad ; \end{array}\right] \;=\; \sum_{n=-\infty}^{\infty} \frac{[a_1,a_2,\dots,a_r;q]_n\,z^n}{[b_1,b_2,\dots,b_r;q]_n} \qquad (1.2)$$

where $\left|\dfrac{b_1,b_2,\dots,b_r}{a_1,a_2,\dots,a_r}\right| < |z| < 1$ for the convergence of (1.2) and $[a_1,a_2,a_3,\dots a_r;q]_n = [a_1;q]_n[a_2;q]_n\dots[a_r;q]_n$.

Also,

$$[a;q]_{-n} \;=\; \frac{(-)^n\;q^{n(n+1)/2}}{a^n[q/a;q]_n} \quad \text{and} \quad [a;\,q]_\infty \;=\; \prod_{r=0}^{\infty}(1-aq^r).$$